Point vortex dynamics in multiply connected domains and its applications

Takashi Sakajo
Hokkaido University, JST CREST
E-mail: sakajo@math.sci.hokudai.ac.jp

Recent numerical studies indicate that the vortex-boundary interaction is a fundamental mechanism to realize an efficient flight of insects[5] and a slow falling of plant seeds[4]. In slightly viscous fluids, vortex structures are generated due to the formation of boundary layers. They begin interacting with the boundaries and create additional forces acting on the boundaries. Thus theoretical study of such vortex-boundary interaction is a mathematical challenge.

Let us consider the incompressible and inviscid fluid flow in two-dimensional multiply connected domains, which is regarded as a simple mathematical model to represent biofluids such as insect-flights, fish swimming and falling leaves. We further assume that the vorticity is concentrated in a discrete point, which is called *a point vortex*. The effect of viscosity is neglected in the point-vortex model. On the other hand, however, since the circulation, which is the strength of the point vortex, is a conserved quantity along the path of a fluid particle according to Kelvin's circulation theorem, we have only to investigate the evolution of point vortices that exist at the initial moment, which makes the theoretical study of vortex-boundary interactions much easier.

In the present talk, I will provide a mathematical formulation to describe the interactions between boundaries and point vortices in multiply connected domains. We first consider a canonical multiply connected domain, called *circular domains*, inside the unit circle containing circular boundaries. The equation of motion for point vortices in the circular domains has been provided by Sakajo[6] based on an explicit analytic representation of the Green function due to Crowdy and Marshall[3]. For a given multiply connected domain, one can derive the equation for point vortices from the canonical equation by constructing a conformal mapping from the target domain to a canonical circular domain. Since the motion of point vortices is not conformally invariant, we need an explicit representation of the conformal mapping. It is usually difficult to express the conformal mapping analytically, so we make use of a numerical conformal mapping technique based on the particle charge simulation methods for the Laplace equation[1, 2].

Moreover, we show an application of the present mathematical formulation to finding stationary configurations of point vortices behind two parallel plates in the presence of a uniform flow, which would be a mathematical model for a wind power generator with vertical blades. We also give another application of the numerical conformal mapping technique to construct the complex potentials in multiply connected channels[7].

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